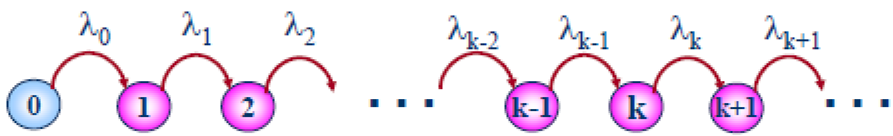
Incoming traffic and service time characterization

Lecture 3

Pure Birth System



- Assumption
  - $\mu_k = 0$ for all k
  - $\lambda_k = \lambda$ for all k
  - The system begins at time t_0 with 0 member

$$p_k(0) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

- $dp_0(t)/dt = -\lambda_0 p_0(t) + \mu_1 p_1(t)$
  - ➔ $dp_0(t)/dt = -\lambda p_0(t)$
- $dp_k(t)/dt = -(\lambda_k + \mu_k)\, p_k(t) + \mu_{k+1} p_{k+1}(t) + \lambda_{k-1} p_{k-1}(t)$
  - ➔ $dp_k(t)/dt = -\lambda p_k(t) + \lambda p_{k-1}(t)$

- Solution for $p_0(t)$
  - ➔ $p_0(t) = e^{-\lambda t}$

- For $k = 1$

  ➔ $dp_1(t)/dt = -\lambda p_1(t) + \lambda p_0(t)$

  $= -\lambda p_1(t) + \lambda e^{-\lambda t}$

  ➔ $p_1(t) = \lambda t e^{-\lambda t}$

- For $k \geq 0,\ t \geq 0$

  ➔ $p_k(t) = \dfrac{(\lambda t)^k}{k!} e^{-\lambda t}$ **Poisson Distribution**

- The arrival of customers

λ = the average rate that customer arrives

Pk(t) = Prob. that *k arrivals occur during (0,t)*

K = # of arrivals in the interval *t*

The average # of arrivals in an interval *t,*

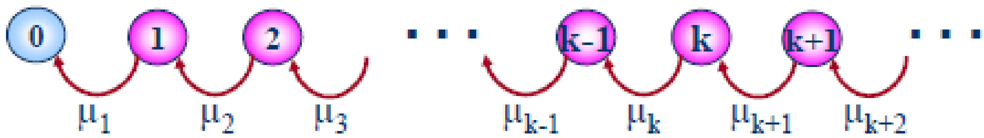
- E[K] = →

$$E[K] = \sum_{k=0}^{\infty} k\, p_k(t) = e^{-\lambda t} \sum_{k=0}^{\infty} k \frac{(\lambda t)^k}{k!}$$

$$= e^{-\lambda t} \sum_{k=1}^{\infty} \frac{(\lambda t)^k}{(k-1)!}$$

$$= e^{-\lambda t} \lambda t \sum_{k=0}^{\infty} \frac{(\lambda t)^k}{k!}$$

$$= \lambda t$$



- Assumption
  - $\mu_k = \mu \geq 0$ for all k
  - $\lambda_k = 0$ for all k
  - The system begins with N members
  - $k = 1,2,3,\ldots,N$

$$p_k(t) = \frac{(\mu t)^{N-k}}{(N-k)!} e^{-\mu t} \qquad 0 < k \leq N$$

$$\frac{dp_0(t)}{dt} = \frac{\mu(\mu t)^{N-1}}{(N-1)!} e^{-\mu t} \qquad k = 0$$

Erlang Distribution

Question Sample

A rural telephone exchange normally experiences four call originations per minute. What is the probability that exactly eight calls occur in an arbitrarily chosen interval of 30 seconds?

Solution

$\lambda = 1/15$ calls per second

When $t = 30$ s, $\lambda t = 2$. Therefore, the probability of exactly eight arrivals is given by

$$P_8(30) = \frac{2^8 e^{-2}}{8!} = 0.00086$$

Question Sample

A switching system serves 10,000 subscribers with a traffic intensity of 0.1 E per subscriber. If there is a sudden spurt in the traffic, increasing the average traffic by 50%, what is the effect on the arrival rate?

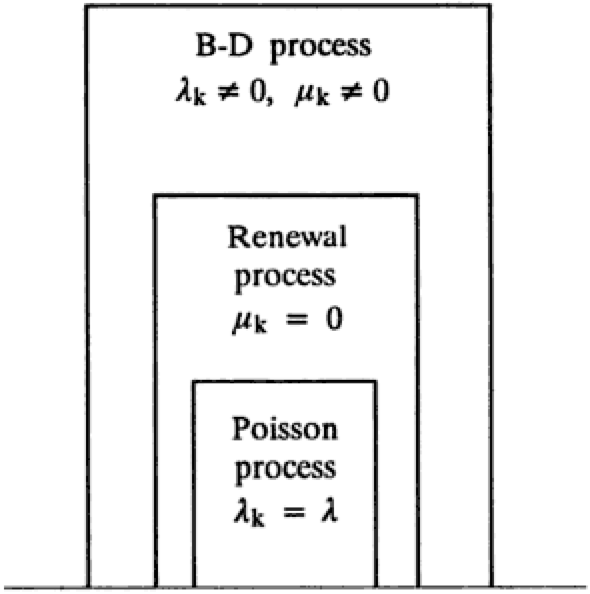
Solution Number of active subscribers during

(a) normal traffic = 1000, (b) increased traffic = 1500

Number of available subscribers for generating new traffic during

(a) normal traffic = 9000, (b) increased traffic = 8500

$$\text{Change in the arrival rate} = \frac{500}{9000} \times 100 = 5.6\%$$



Relationship among different Markov processes.

Venn diagram, we may describe the Poisson process as follows:

1. A pure birth process with constant birth rate
2. A birth-death process with zero death rate and a constant birth rate
3. A Markov process with state transitions limited to the next higher state or to the same state, and having a constant transition rate.

In real life, Poisson process occurs very often. Examples include:

1. Number of telephone calls arriving at an exchange
2. Number of coughs generated in a medical ward by the patients
3. Number of rainy days in a year
4. Number of typing errors in a manuscript
5. Number of bit errors occurring in a data communication system.

The probability of finding k busy channels at any instant of time is merely the probability that there are k arrivals during the interval time tm immediately preceding the instant of observation given by:

$$P_k(t_m) = \frac{(\lambda t_m)^k \exp(-\lambda t_m)}{k\,!}$$

we proposed the use of a birth-death process for modelling a telecommunication system. In this section, we have shown that the call arrivals can be modelled as a Poisson process and we describe how the holding times of the calls have an exponential distribution when the system is modelled as a B-D process. A constant holding time distribution is useful for modelling exchange functions.